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THEORETICAL AND MATHEMATICAL PHYSICS

Self-Similar Distribution in "Giant" Magnetic Flux Creep

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Abstract—The problem of magnetic field penetration into the half-space is considered in parallel geometry in an external magnetic field increasing with time in accordance with the law $B(0, t, \tau_0) = B_{c_1}(1 + t/\tau_0)^m$, $m \ge 0$, $t \ge 0$ (τ_0 is the time of magnetic flux redistribution and B_{c_1} is the lower critical field). It is assumed that the flow of vortices is thermally activated in the "giant" creep mode (i.e., for weak pinning creep and high temperatures). A model equation is derived for describing the magnetic flux evolution. Analytic formulas are obtained for the depth and velocity of magnetic field penetration. It is shown that the giant creep regime is stable for $0 \le m \le 1/2$.

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Let us consider the problem of magnetic flux penetration into a high-T_c superconductor occupying the half-space $x \ge 0$ in parallel geometry $\mathbf{B} \parallel \mathbf{e}_z$, $\mathbf{E}$, $\mathbf{J} \parallel \mathbf{e}_y$, and $\mathbf{v} \parallel \mathbf{e}_x$, where $\mathbf{e}$ is a unit vector, $\mathbf{E}$ is the electric field, $\mathbf{J}$ is the current density, $\mathbf{v}$ is the velocity of vortices during magnetic flux creep [1],

$$v = v_0 \exp(-(cU_0 - BJV_c d_p)/cT), \quad (1)$$

v_0 is the microscopic velocity of vortices, c is the velocity of light, d_p is the mean activation distance for bundles of vortex filaments, U_0 is the pinning barrier, V_c the activation volume, in which the vortex lattice is deformed under the action of potential U_0, and T is the temperature. In accordance with formula (1), the velocity of vortices depends on parameter $\mu = U_0/T$, where $\mu \gg 1$ for ordinary rigid superconductors and the value of μ is several orders of magnitude smaller for high-T_c superconductors. This can be explained by the small coherence length, which leads to a low pinning barrier and high temperatures. This phenomenon, which was discovered by Yeshurun and Malozemoff [2], is called the "giant" magnetic flux creep.

Relation (1) can be written in dimensionless form,

$$v = v_0 e^{-\mu(1 + \kappa b b_x)}\left(\kappa = \frac{1}{\beta}\frac{B_{c_2}^2}{H_c^2}\frac{V_c d_p}{\lambda \zeta_{\parallel} a_0^2}\right), \quad (2)$$

where β is a constant proportional to the number of vortex filaments in the bundle [3], H_c is the thermodynamic critical field, B_{c_2} is the upper critical field, $\zeta_{\parallel}$ is the coherence length parallel to a vortex filament, a_0 is the Abrikosov lattice constant, λ is the penetration depth in the Meissner phase, and $x \longrightarrow x/\lambda$.

We assume that the condition $\kappa\mu \ll 1$ is satisfied. Then we obtain from Eq. (2) and Maxwell equations

$$E = Bv/c, \quad c^{-1}\partial_t B = -\partial_x E$$

(c is velocity of light) in the linear approximation the equation

$$b_{t'} + \sigma^{-1} b_{x'} = (b^2 b_{x'})_{x'}, \quad (3)$$

$$B(0, t, \tau_0) = B_0(1 + t/\tau_0)^m, \quad t \ge 0, \quad m \ge 0, \quad (4)$$

where $t' = t/\tau_0$, $x' = x/\lambda$, $D = (\tau_0 v_0/\lambda)k\mu e^{-\mu}$, τ_0 is the time over which the system passes to the scaling mode [4, 5]. Condition (4) can be written in dimensionless form

$$b(0, t') = b_0(1 + t')^m, \quad (5)$$

where $b_0 = B_0/B_{c_2}$, $B_{c_1} < B < B_{c_2}$. The field increases in accordance with law (5) over a finite time interval, after which the field stabilizes or decreases so that

$$\partial_{t'} b(0, t') \longrightarrow \infty \quad \text{for} \quad t' \longrightarrow \infty.$$

In this case, problem (3), (5) with the initial distribution $b(x', 0) = b_0$ simulates the magnetic flux evolution in the superconductor, which is preliminarily cooled in zero magnetic field (see p. 1352 in [5]). In the subsequent analysis, we will omit primes in the notation.

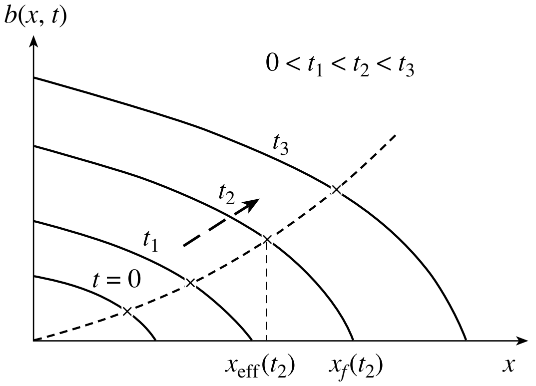

Fig. 1. Typical profile of the magnetic field amplitude for an exponential increase in the external magnetic field in the giant creep regime; x_f is the coordinate of the front, x_{eff} is the effective coordinate of the front, after the attainment of which the amplitude decreases by a factor of two.

Let the solution of problem (3), (5) have the form [6]

$$b(x, t) = b_0(1+t)^{\alpha}\phi(\eta, t), \quad \eta = x(1+t)^{-\delta}, \quad \alpha > 0, \quad \delta > 0. \tag{6}$$

Then substitution of Eq. (6) into (3) leads to the equation

$$\phi_t - t^{-1}\sigma^{-1}\eta\phi_\eta + \alpha t^{-1}\phi = b_0^2 t^{2\alpha-2\delta}(\phi^2\phi_\eta)_\eta - \sigma^{-1}t^{-\delta}\phi_\eta \ (t \longrightarrow 1+t). \tag{7}$$

For $t \gg 1$ and $\alpha = \delta$, Eq. (7) leads to the equation

$$\phi_t = (\phi^2\phi_\eta)_\eta \ (t \longrightarrow b_0^2 t), \tag{8}$$

for which we consider the boundary condition

$$\phi(0, t) = (1+t)^p, \quad p \geq 0. \tag{9}$$

The solution to problem (8), (9) has the form [6]

$$\phi_A(\eta, t) = (1+t)^p\theta(\zeta), \quad \zeta = \eta/(1+t)^{(1+2p)/2}, \tag{10}$$

where function $\theta(\zeta)$ behaves as shown in Fig. 1. For $p = 1/2$, we obtain

$$\theta(\zeta) = (1-\sqrt{2}\zeta)_+^{1/2}.$$

By virtue of Eqs. (6) and (10), we find that

$$b(x, t) = b_0(1+t)^{\alpha+1/2}(1-\sqrt{2}x/(1+t)^{\alpha+1})_+^{1/2}. \tag{10'}$$

For $p \neq 1/2$, we have

$$b(x, t) = b_0(1+t)^m\theta(\zeta(x, t, m)), \quad m = \alpha + p. \tag{11}$$

For

$$\zeta = \zeta_{\text{eff}} = \theta^{-1}(1/2)$$

($\theta^{-1}(\cdot)$ is a function reciprocal to ($\theta(\cdot)$)), the effective depth of magnetic field penetration is

$$x_{\text{eff}}(t) = \zeta_{\text{eff}}(1+t)^{m+1/2},$$

while the velocity of magnetic flux propagation is

$$v_{\text{eff}}(t) = \zeta_{\text{eff}}(m+1/2)(1+t)^{m-1/2}.$$

We write the relations derived above in dimensional form:

$$x_{\text{eff}}(t) = \zeta_{\text{eff}}\lambda(1+Dt/\tau_0)^{m+1/2}, \tag{12}$$

$$v_{\text{eff}}(t) = \zeta_{\text{eff}}\lambda(m+1/2)D_{\tau_0}^{-1}(1+Dt/\tau_0)^{m-1/2}. \tag{13}$$

Let us suppose that $\tau_0 v_0/\lambda = 1$. For $\lambda \sim 10^{-5}$ cm and $\tau_0 \sim 10^{-1}$–10^{-4} s (experimental values for YBaCuO [4]), we obtain an estimate $10^{-4} < v_0 < 10^{-1}$ cm/s, which is comparable to the classical creep velocity [7–9]. Then Eq. (12) leads to the relation

$$x_{\text{eff}}(t) = \zeta_{\text{eff}}\lambda(1+\kappa\mu e^{-\mu}\tau_0^{-1}t)^{m+1/2},$$

where $\tau_0 \sim 10^0$–10^4 s and $\partial_t B(0, t) \sim 10^{-3}$–$10^{-6}$ T/s are chosen in accordance with the experiment from [4]. It follows hence that the penetration depth for $t \sim \tau_0$ is

$$x_{\text{eff}}(\tau_0) = \zeta_{\text{eff}}\lambda(1+\kappa\mu e^{-\mu})^{m+1/2},$$

which, in turn, leads to the relation

$$v_{\text{eff}}(\tau_0) = \zeta_{\text{eff}}\lambda(m+1/2)\lambda\mu e^{-\mu}\tau_0^{-1}(1+\kappa\mu e^{-\mu})^{m-1/2}. \tag{14}$$

This relation for $m = 1/2$ gives the equality

$$v_{\text{eff}}(\tau_0) = \zeta_{\text{eff}}\lambda\kappa\tau_0^{-1}\mu e^{-\mu},$$

which leads to the estimate

$$\zeta_{\text{eff}}\lambda\kappa\mu e^{-\mu} < v_{\text{eff}}(\tau_0) < 10^4\zeta_{\text{eff}}\lambda\kappa\mu e^{-\mu} \text{ cm/s} \quad (\mu = U_0/T),$$

which gives the values of velocity in the conventional creep regime.

The graphic representation of analytic relations (12) and (13) is given in Figs. 2 and 3, respectively, which show that the magnetic flux penetration depth and velocity decrease (increase) for $\mu > 1$ upon an increase in the pinning barrier (decrease in temperature). In the viscous flow regime, analogous temperature dependence was obtained, for example, in [10] for the model of the critical state in a plane-parallel plate. The case $\mu < 1$ is realized at temperatures close to critical temperature T_c. In this case, we must take into account the temperature dependence of pinning energy [1]. In such a situation, thermal energy $k_B T$ ($k_B = 1$ is the Stephan–Boltzmann constant) is higher than the activation barrier energy, which leads to a thermally activated increase in the magnetic field penetration depth. This velocity attains its maximal value when the corresponding (thermal and activation) energies are equal. Subsequently, for $\mu > 1$, the thermally activated penetration of

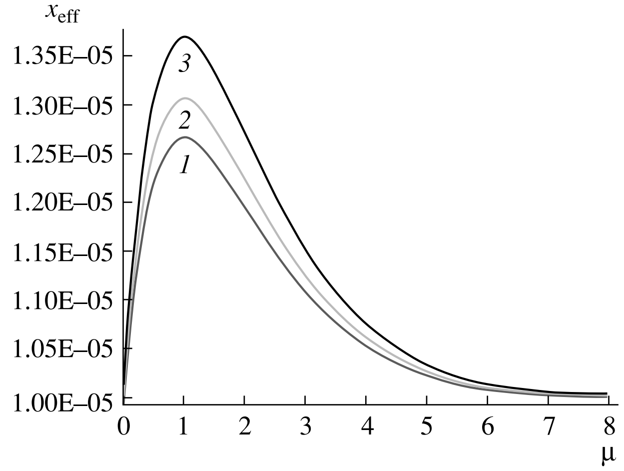

Fig. 2. Dependence of the velocity of magnetic flux penetration during giant creep in an increasing external field on parameter $\mu = U_0/T$ for m = 1/4 (*1*), 1/3 (*2*), and 1/2 (*3*).

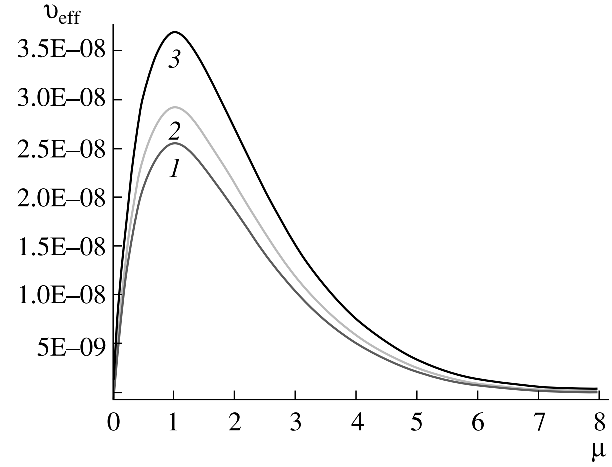

Fig. 3. Dependence of the penetration depth on parameter μ for m = 1/4 (*1*), 1/3 (*2*), and 1/2 (*3*).

the magnetic flux follows the standard scenario. The experiments confirming the qualitative behavior of the curves in Figs. 2 and 3 for $\mu \le 1$ have not been carried out.

Figure 1 shows the magnetic field distribution during creep in an increasing external field, which has a simple physical meaning. Indeed, it follows from Eq. (3) that the magnetic field distribution in a plate for $\sigma \longrightarrow 0$ is constant and, hence, depends only on the boundary conditions, which is confirmed by the analytic representation of solution (10'). For $\sigma > 0$, the flux begins to penetrate the superconductor in a thermally activated manner. This process (creep) is facilitated by an increase in the external field, which precisely leads to the curve depicted in Fig. 1.

An analogous result for ordinary creep during a linear increase in the magnetic field with time was obtained in [11] for half-space. An analogous theoretical result in the viscous flow mode for vortices was obtained in [12] using the model of the critical state for oxide high-T_c superconductors. For exponent $m > 1/2$ in the pump rate, we pass consecutively to the viscous flow regime and then, for $B \gg B_{c_1}$, to the emergence of a certain analog of a dendrite structure [13]. Such a situation, which corresponds to experiment from [14], is not investigated here.

We consider an example of exact solution of Eq. (13) for $m = 1/2$:

$$b(x, t) = [2(\lambda - \sigma^{-1})(\lambda t - x)_+]^{1/2}. \qquad (15)$$

Solution (15) satisfies the boundary condition $b(0, t) \propto b_0 t^{1/2}$; the flux penetration velocity is $v_{eff} = \lambda$ for $\lambda > \sigma^{-1}$. In this case, convection does not affect the velocity, but reduces the field penetration depth. For $\sigma \gg 1$ (i.e., when the activation energy for vortex filaments is high, which corresponds to the model of vortex glass for a very high pinning barrier), solution (15) qualitatively coincides with the solution from [5] and is close (in form) to the curve describing the magnetic field distribution, which was obtained in [11] for $m = 1$. Using Eq. (15), we can determine the current density

$$j(x, t) = \rho[(\lambda t - x)_+]^{-1/2},$$

where $\rho = 2^{-1/2}(\lambda - \sigma^{-1})^{1/2}$, which leads to the relaxation law for the total current density

$$\langle j(t)\rangle = x_{eff}^{-1}(t)\int_0^{x_{eff}(t)} j(s, t)ds \propto (1 + t)^{-1/2}. \qquad (16)$$

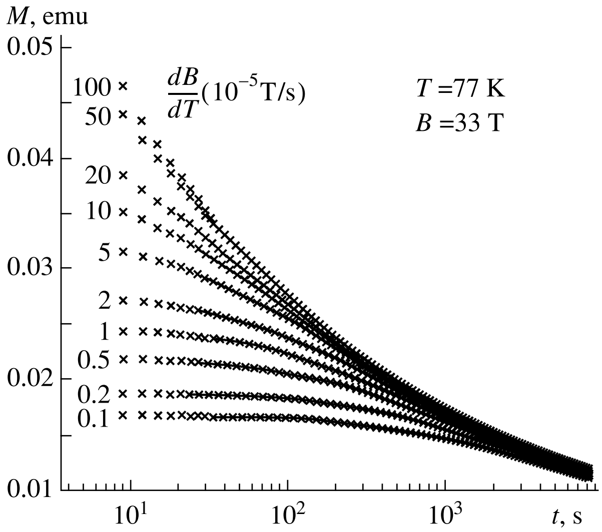

Fig. 4. Relaxation curves $M(\ln t)$ for T = 77 K and B = 3 T for various rates $B_t \times 10^{-5}$ T/s of pumping by an external magnetic field (relaxation is logarithmic) [4].

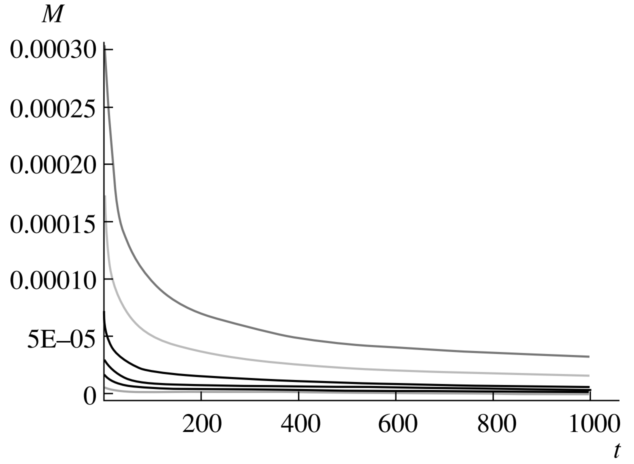

Fig. 5. Magnetization curves in an increasing magnetic field during giant creep over long time intervals for a superconducting half-space in a parallel external field. Magnetization curves correspond to the relevant curves in Fig. 4 for various rates $B_t \times 10^{-5}$ T/s of pumping by an external magnetic field. The upper and lower curves correspond to pump rates $B_t = 100$ and 1 T/s, respectively. The curves in Fig. 4 for $B_t < 1$ T/s correspond to nearly coinciding curves in the figure.

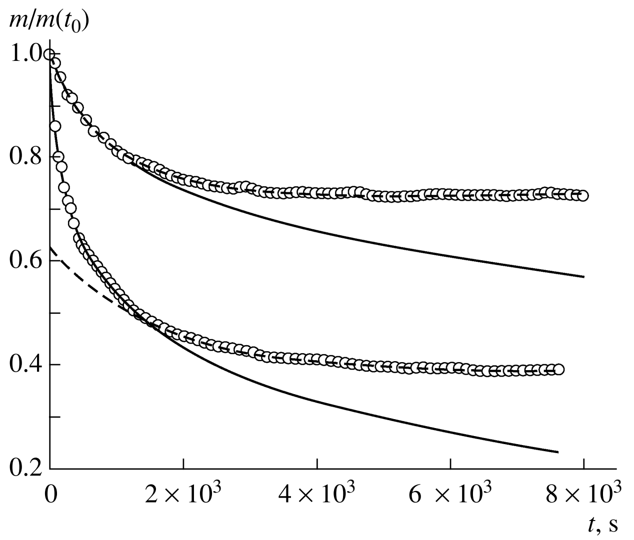

Fig. 6. Magnetization curves at $T = 4.5$ K for $\mu_0 = 1.5$ T (○) and 1.7 T (□). Solid (dashed) curves decrease in accordance with logarithmic (exponential) law for short (long) time intervals. The crossover occurs at critical values $t_{cr} = 1500$ s for $\mu_0 H = 1.5$ T and $t_{cr} = 800$ s for $\mu_0 H = 1.7$ T, where μ_0 is the permeability of the medium and H is the magnetic field. The superconductor has the form of a plate in a parallel external magnetic field [16].

From total current $\langle j(t)\rangle$, we determine the magnetic moment

$$M(t) = \int_0^a x\langle j(t)\rangle dx = \langle j(t)\rangle \frac{a^2}{2}, \qquad (17)$$

where $t^* = x_{\text{eff}}^{-1}(a)$ and $2a$ id the plate thickness $x_{\text{eff}}^{-1}(\cdot)$ is a function reciprocal to $x_{\text{eff}}(\cdot)$, and

$$t^* = \frac{\tau_0}{D}\left[\left(\frac{a}{\zeta_{\text{eff}}\lambda}\right)^{1/(m+1/2)} - 1\right].$$

Formula (17) is applicable for comparison with the experiment illustrated in Fig. 4 [4] for a superconducting plate in a parallel magnetic field. For a half-plate, formula (17) can be generalized as follows:

$$M(t) = \langle j(t)\rangle \frac{x_{\text{eff}}^2}{2} \propto (1+t)^{2m+1/2}.$$

It should be noted that the magnetic field distribution (for a plate) in a constant external field has the form [3]

$$B(x,t) = B_0 - x\frac{4\pi}{c}J_c\left[\frac{T}{T_*}\ln\left(\text{const} + \frac{t}{\tau_0}\right)\right]^{-1/\beta} \qquad (18)$$
$$(0.2 < \beta \le 0.5).$$

It follows from this relation that magnetization relaxation with time follows the law

$$M(t) \sim [\ln(\text{const} + t)]^{-1/\beta},$$

where β depends on the dimension of bundles of vortex filaments, temperature, applied magnetic field, transport current, and initial conditions. Relation (18) differs from the law $\ln t$ for flux creep.

In the general case, we obtain the relation for the current density

$$j(x,t) = -kb_0(1+t)^{-1/2}\theta'(\zeta(x,t,m)), \qquad (19)$$

where $k = cB_{c_2}/(4\pi J_c\lambda)$. Relation (19) for $p = 1/2$ and $t \gg \tau_0$ leads to the asymptotic form

$$\langle j(t)\rangle \propto (1+t)^{-1/2}.$$

For $p \neq 1/2$, if

$$\theta'(x,t) \sim \text{const} + (1+t)^{-\nu}, \quad \nu > 1/2,$$

asymptotic form (16) is conserved. The magnetization curve (17) plotted in accordance with formula (16) is depicted in Fig. 5. Comparison of the curves in Fig. 5 and Fig. 4 from [4] shows that the magnetization profiles coincide satisfactorily for long time periods. Instead of logarithmic relaxation, power relaxation takes place in this case, leading to a slower decrease in magnetization with time. In [16], a crossover was observed (at a fixed temperature) from a logarithmic to an exponential law of decrease in magnetization during thermally activated magnetic flux creep. In weak fields, the behavior of magnetization is identical to the behavior of the curves on initial time intervals depicted in Fig. 6. It should be noted that the experiment [4] corresponds to a fairly high ($U_0 \gg T$) activation energy of pinning barrier (ordinary creep), and experiment depicted in Fig. 6 from [16] is closer to the "giant" creep conditions.

In conclusion, we can formulate the following results:

(i) it is shown that the giant creep regime in a weakly increasing external field may be stable if the pump rate is low ($m \leq 1/2$);

(ii) a weakly increasing external field may lead to a creep velocity which is an order of magnitude (or even more) higher than the velocity of giant creep;

(iii) analytical formulas are given for the magnetic flux penetration depth and velocity, which depend on index $m > 0$;

(iv) specific numerical estimates are given, which are in agreement with the available experimental estimates obtained using the magnetooptical methods (see review [15]);

(v) it is shown that magnetization relaxation in giant creep follows a power law in contrast to the logarithmic relaxation in the case of ordinary creep, which leads to a slower relaxation to the equilibrium position.

The latter statement can be explained by a higher velocity of magnetic flux penetration as compared to the creep velocity for traditional superconductors.

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