Global existence and asymptotic stability for an elliptic-parabolic free boundary problem: an application to a model of tumor growth, *Indiana Univ. Math. J.*, 52, № 5 (2003), 1265-1304. (A. Friedman).

Abstract

We consider a free boundary problem for a coupled system consisting of an elliptic equation $\Delta p+\mu(\sigma-\tilde{\sigma})=0$ $(\mu>0)$ for p and a parabolic equation for σ. The problem is motivated by a model of tumor growth whereby p represents the pressure of the proliferating cells and σ is the concentration of nutrients. On the boundary $\Gamma(t)$ of the tumor region, p is equal to the surface tension, and the flux of p is equal to the normal velocity of $\Gamma(t)$. In the case $\mu=0$, the system decouples into a Hele-Shaw problem for p and a standard parabolic equation for σ. For the Hele-Shaw problem it is known that there are stationary radially symmetric solutions and each one is asymptotically stable in the following sense: If we take for initial data a small perturbation of a radially symmetric solution, then the corresponding Hele-Shaw problem has a unique global solution and its free boundary converges to a sphere as $t\to\infty$. In this paper, we prove a similar result for the coupled elliptic-parabolic problem provided μ is small. The asymptotic stability result is generally false if μ is not small.